Centralized Two-Way AF MIMO Multiple Dispersed Relay Network

Kanghee Lee, Jie Yang, Hyuck M. Kwon, Hyuncheol Park, and Yong H. Lee

***Abstract*—This paper considers a two-way amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay network consisting of two sources (each with multiple antennas) and multiple physically dispersed single-antenna relays. A central station finds the optimum relay amplifying matrices and transmit/receive beamforming vectors explicitly and iteratively under the transmit power constraints at the sources and the relays. The minimum mean square error (MMSE) criterion is used. Numerical results show that the proposed scheme outperforms the existing two-way AF MIMO network schemes in [1], [2].**

***Index Terms*—Two-way, amplify-and-forward (AF), minimum mean square error (MMSE), beamforming, power constraint.**

I. Introduction

Recently, various *two-way* wireless relaying schemes have been studied to improve spectral efficiency [1]–[3]. Arti *et al.* in [1] studied a beamforming and combining scheme for a two-way amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay network consisting of two sources and a single relay, where both sources and relay have multiple antennas. The maximum ratio transmission and maximum eigenvalue principles were used in [1]. This current paper assumes that each relay does not know the entire channel coefficients, but they report their locally available channel coefficients to a cloud radio access network (CRAN) so that the CRAN can compute the optimum diagonal AF relay amplifying matrix and forward its i-th diagonal element to the i-th relay. Hence, we call the proposed scheme the "centralized scheme for physically dispersed relays (CSPDR)."

The two-way AF MIMO wireless relay system with a multi-antenna single relay between two multi-antenna sources has also recently attracted attention [1]. However, the system with multiple dispersed single-antenna relays between two multi-antenna sources has not been studied much in the literature except for a few papers, e.g., [2], where a *diagonal* relay amplifying matrix and beamforming vectors were presented for the two-way AF MIMO relay network. The minimum mean square arror (MMSE) criterion under the transmit power constraint at two sources and relays was applied to determine them in [2]. As a special case of [2], the two-way AF single-input single-output (SISO) wireless relay system with multiple single-antenna relays between two single antenna sources has been studied in [3]. The relay amplifying matrix was designed under only the relay transmit power constraint in [3].

In this paper, we focus on the CSPDR for the two-way AF MIMO wireless relay network consisting of a source-destination pair with *multiple* antennas and *multiple* relays with a *single* antenna per relay, as done in [2]. The difference between the proposed CSPDR and the existing one in [2] is presented in Section IV. The objective of this paper is to jointly determine MMSE-based AF relay amplifying matrices at the relays and transmit/receive beamforming vectors at the sources/destinations by imposing the transmit power constraints at the two sources and relays, independently and individually. This paper provides an iterative algorithm that can solve the optimal transmit/receive beamforming vectors and relay amplifying matrix.

The main contributions of this paper are summarized as follows:

- This paper has solved the optimal diagonal AF relay amplifying matrices and optimal transmit/receive beamforming vectors at the sources/destinations explicitly and iteratively under source and relay transmit power constraints simultaneously, although the existing references [1]–[3] have not.
- The symbol error rate (SER) performance with the proposed CSPDR outperforms, e.g., 6.5 dB gain at SER $= 10^{-3}$ shown in Fig. 4, the existing one in [2]. In particular, the number of iterations is less than 15 for most practical cases. Hence, the proposed solutions are more efficient than the exhaustive search in [2].
- The relay transmit power p_r in this paper is constrained to be 1, regardless of either M (number of transmit antennas at source) or N (number of relays), whereas the relay transmit power in [1] increases as M or N

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This work was partly sponsored by the 2014 Air Force Summer Faculty Fellowship Program, the Army Research Office under DEPSCoR ARO Grant W911NF-08-1-0256, NASA under EPSCoR CAN Grant NNX08AV84A, The Cross-Ministry Giga KOREA Project of The Ministry of Science, ICT and Future Planning, Korea, [GK13N0100, 5G mobile communication system development based on mmWave], and by the Defense Acquisition Program Administration (DAPA) and the Agency for Defense Development (ADD) of Korea under Grant UTRC-KAIST.

IEEE Transactions on Vehicular Technology Year: 2015 , Volume: 64

increase. In addition, the proposed scheme shows much better performance, e.g., 4.1 dB at SER $= 10^{-2}$ in Fig. 5, than the existing one in [1] with the normalized relay transmit power.

The remainder of this paper is organized into four sections. Section II describes the system model and data transmission strategies applied. Section III provides the CSPDR with a set of optimal relay amplifying matrix and transmit/receive beamforming vectors under transmit power constraints at both the sources and the relays. In addition, the optimal relay amplifying matrix is determined using the eigen beamforming (EB) vectors. Section IV shows the average simulation MMSE, bit error rate (BER), and sum rate results using the proposed CSPDR. Finally, Section V concludes the paper.

Notation: Matrices, vectors, and scalars are denoted, respectively, by uppercase boldface, lowercase boldface, and italic characters (e.g., $\mathbf{A}$, $\mathbf{a}$, and a). The inverse, transpose, trace, and Hermitian of $\mathbf{A}$ are denoted, respectively, by $\mathbf{A}^{-1}$, $\mathbf{A}^T$, $tr(\mathbf{A})$, and $\mathbf{A}^H$. An $N \times N$ identity and diagonal matrix are denoted, respectively, by $\mathbf{I}_N$ and diag$(a_1, \cdots, a_N)$. Notations $|a|$, $||\mathbf{a}||$, and $||\mathbf{A}||_F$ denote the absolute value of a for any scalar, 2-norm of $\mathbf{a}$, and Frobenius-norm of $\mathbf{A}$, respectively. The expectation and Hadamard product operators are denoted by $E[\,\cdot\,]$ and $\odot$, respectively. The diag$(\mathbf{D})$ is denoted by an $N \times 1$ column vector $\mathbf{d}$ that consists of the diagonal elements of the $N \times N$ diagonal matrix $\mathbf{D}$. The diag$(\mathbf{d})$ is denoted by an $N \times N$ diagonal matrix whose diagonal entries are elements of $\mathbf{d}$.

II. System Model

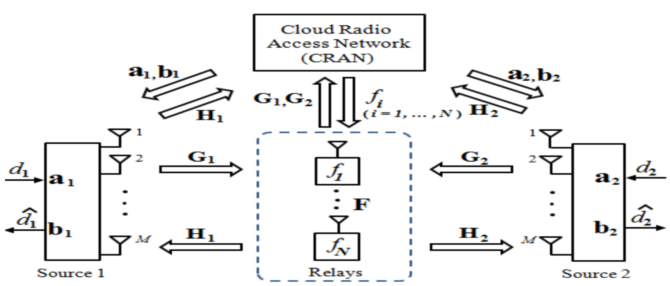

Fig. 1. Two-way AF MIMO relay network under transmit power constraints at sources and relays.

Figure 1 shows a block diagram of the proposed CSPDR that will be studied under power constraints at two sources, $\mathbb{S}_1$ and $\mathbb{S}_2$, and the relays. During the first (or odd) time slot, $\mathbb{S}_1$ and $\mathbb{S}_2$ transmit, respectively, their data $d_1 \in \mathbf{C}^{1\times 1}$ and $d_2 \in \mathbf{C}^{1\times 1}$ to the N relays through their beamforming vectors $\mathbf{a}_1 \in \mathbf{C}^{M\times 1}$ and $\mathbf{a}_2 \in \mathbf{C}^{M\times 1}$. Each relay has a single antenna element, and each source has M number of antenna elements. During the second (or even) time slot, the i-th relay amplifies the received signal with the optimum AF relay amplifying coefficient $f_i \in \mathbf{C}^{1\times 1}$, and forwards it to both $\mathbb{S}_1$ and $\mathbb{S}_2$, $i = 1, \cdots, N$. Note that the AF relay amplifying matrix F is diagonal, as are the ones in [2], [3]. Then, $\mathbb{S}_1$ and $\mathbb{S}_2$ receive the AF signal with their receive beamforming vectors $\mathbf{b}_1 \in \mathbf{C}^{1\times M}$ and $\mathbf{b}_2 \in \mathbf{C}^{1\times M}$, respectively, and $\mathbb{S}_i$ decodes d_j from source $\mathbb{S}_j$ after subtracting its own data d_i, $i \neq j$. This transmit-and-receive processing continues within a quasi-static frame of L time slots, $L \gg 1$. Let $\mathbf{G}_1 \in \mathbf{C}^{N\times M}$, $\mathbf{G}_2 \in \mathbf{C}^{N\times M}$, $\mathbf{H}_1 \in \mathbf{C}^{M\times N}$, and $\mathbf{H}_2 \in \mathbf{C}^{M\times N}$ denote, respectively, the complex channel coefficient matrices from $\mathbb{S}_1$ to N relays, from $\mathbb{S}_2$ to N relays, from N relays to $\mathbb{S}_1$, and from N relays to $\mathbb{S}_2$. The elements of all backward $\mathbf{G}_i$ and forward channels $\mathbf{H}_i$ are independent and identically distributed (i.i.d.), zero-mean and unit-variance circular complex Gaussian, and quasi-static Rayleigh fading. At the beginning of every quasi-static Rayleigh frame, $\mathbb{S}_1$, $\mathbb{S}_2$, and the N relays transmit their channel coefficient matrices ($\mathbf{H}_1$, $\mathbf{H}_2$) and ($\mathbf{G}_1$, $\mathbf{G}_2$) to the CRAN within a time slot. Then, the CRAN computes the optimum transmit-and-receive beamforming vectors ($\mathbf{a}_1$, $\mathbf{b}_1$), ($\mathbf{a}_2$, $\mathbf{b}_2$), and the optimum relay amplifying coefficient f_i, and forwards them, respectively, to $\mathbb{S}_1$, $\mathbb{S}_2$, and the i-th relay, $i = 1, \cdots, N$. This channel state information (CSI) estimation may take a time slot interval and may cause a slight throughput reduction by $1/L$, but it is neglected in this paper because L can be larger than one in practice.

This paper assumes that the AF relay network is synchronized by a CRAN to whom the individual relay nodes report their received signals and their local CSI. Then, the CRAN acquires and tracks the global synchronization. In addition, the CRAN computes the optimum diagonal relay amplifying matrix and forwards the individual amplifying coefficients to each relay.

The objective of this paper is to find the optimum transmit and receive beamforming vectors ($\mathbf{a}_1$, $\mathbf{b}_1$) and ($\mathbf{a}_2$, $\mathbf{b}_2$), and optimum AF relay amplifying coefficients f_i, $i = 1, \cdots, N$, under the two source power constraints and the relay power constraint.

The receiver beamforming vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ can be represented, respectively, as $\mathbf{b}_1 = \gamma_1^{-1}\hat{\mathbf{b}}_1$ and $\mathbf{b}_2 = \gamma_2^{-1}\hat{\mathbf{b}}_2$, where γ_1^{-1} and γ_2^{-1} are positive scaling factors that will be the same for all antennas at $\mathbb{D}_1$ and $\mathbb{D}_2$, and $\hat{\mathbf{b}}_1$ and $\hat{\mathbf{b}}_2$ are normalized beamforming vectors satisfying $||\hat{\mathbf{b}}_1||^2 = 1$ and $||\hat{\mathbf{b}}_2||^2 = 1$.

The transmitted symbol vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ from $\mathbb{S}_1$ and $\mathbb{S}_2$ are given, respectively, by $\mathbf{s}_1 = \mathbf{a}_1 d_1$ and $\mathbf{s}_2 = \mathbf{a}_2 d_2$ with $E[|d_1|^2] = \sigma_{d_1}^2$, $E[|d_2|^2] = \sigma_{d_2}^2$, $E[||\mathbf{s}_1||^2] = p_{s_1}$, and $E[||\mathbf{s}_2||^2] = p_{s_2}$. The received signal vector $\mathbf{r} \in \mathbf{C}^{N\times 1}$ at the relay inputs can be written as $\mathbf{r} = \mathbf{G}_1\mathbf{s}_1 + \mathbf{G}_2\mathbf{s}_2 + \mathbf{v}_s$, where $\mathbf{v}_s \in \mathbf{C}^{N\times 1}$ is an additive white Gaussian noise (AWGN) vector with zero-mean and covariance matrix $\sigma_{v_s}^2\mathbf{I}_N$.

In the even time slot, the relays transmit their received signals to $\mathbb{S}_1$ and $\mathbb{S}_2$ after multiplying an $N \times N$ diagonal relay amplifying matrix F. (Now, $\mathbb{S}_1$ and $\mathbb{S}_2$ are called by destinations $\mathbb{D}_1$ and $\mathbb{D}_2$, respectively.) Hence, the transmitted signal $\mathbf{x} \in \mathbf{C}^{N\times 1}$ at the relay outputs is given by $\mathbf{x} = \mathbf{F}\mathbf{r}$ with total transmitted power $x^H x = p_r$ used by N relays. Then, receive beamforming vectors $\mathbf{b}_1 \in \mathbf{C}^{1\times M}$ and $\mathbf{b}_2 \in \mathbf{C}^{1\times M}$ are applied, respectively, as $\hat{d}_1 = \mathbf{b}_2\mathbf{y}_2$ and $\hat{d}_2 = \mathbf{b}_1\mathbf{y}_1$.

The estimated data symbol $\hat{d}_1$ and $\hat{d}_2$ can be written, respectively, as

$$\begin{aligned}\hat{d}_1 &= \mathbf{b}_2\mathbf{H}_2\mathbf{F}\mathbf{G}_1\mathbf{s}_1 + \mathbf{b}_2\mathbf{H}_2\mathbf{F}\mathbf{v}_s + \mathbf{b}_2\mathbf{z}_2\\ \hat{d}_2 &= \mathbf{b}_1\mathbf{H}_1\mathbf{F}\mathbf{G}_2\mathbf{s}_2 + \mathbf{b}_1\mathbf{H}_1\mathbf{F}\mathbf{v}_s + \mathbf{b}_1\mathbf{z}_1. \end{aligned} \quad (1)$$

where $\mathbf{z}_1$ and $\mathbf{z}_2$ are AWGN vectors with zero-mean and covariance matrices $\sigma_{z_1}^2\mathbf{I}_M$ and $\sigma_{z_2}^2\mathbf{I}_M$, respectively. The self interference is cancelled at each destination.

III. AF Beamforming Strategies

The sum of the MSE between d_1 and $\hat{d}_1$ and the MSE between d_2 and $\hat{d}_2$ will be minimized under the transmit power constraints at the sources and relays. The cost function $J(\mathbf{F}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2)$ is defined, using the MMSE criterion [5], as

$$J(\mathbf{F}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2) \triangleq E[|\hat{d}_1 - d_1|^2] + E[|\hat{d}_2 - d_2|^2]. \quad (2)$$

The transmit power usage at the two sources and the N relays can be expressed as $p_{s_1} = \sigma_{d_1}^2||\mathbf{a}_1||^2$, $p_{s_2} = \sigma_{d_2}^2||\mathbf{a}_2||^2$, and $p_r = \sigma_{d_1}^2||\mathbf{F}\mathbf{G}_1\mathbf{a}_1||^2 + \sigma_{d_2}^2||\mathbf{F}\mathbf{G}_2\mathbf{a}_2||^2 + \sigma_{v_s}^2||\mathbf{F}||_F^2$. For notational convenience, the cost function $J(\mathbf{F}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2)$ in (2) is henceforth simply stated as $\mathcal{J}(\mathbf{F})$. Hence, $\mathcal{J}(\mathbf{F})$ in (2) can be written as

$$\begin{aligned}\mathcal{J}(\mathbf{F}) = &\sigma_{d_2}^2\gamma_1^{-2}\mathbf{f}^H(\mathbf{K}_1^T \odot \hat{\mathbf{K}}_2)\mathbf{f} + \sigma_{v_s}^2\gamma_1^{-2}\mathbf{f}^H(\hat{\mathbf{K}}_2 \odot \mathbf{I}_N)\mathbf{f}\\ &-\sigma_{d_2}^2\gamma_1^{-1}\hat{\mathbf{k}}_3^H\mathbf{f} - \sigma_{d_2}^2\gamma_1^{-1}\mathbf{f}^H\hat{\mathbf{k}}_3 + \sigma_{z_1}^2\gamma_1^{-2}\hat{\mathbf{b}}_1\hat{\mathbf{b}}_1^H + \sigma_{d_2}^2\\ &+\sigma_{d_1}^2\gamma_2^{-2}\mathbf{f}^H(\mathbf{B}_1^T \odot \hat{\mathbf{B}}_2)\mathbf{f} + \sigma_{v_s}^2\gamma_2^{-2}\mathbf{f}^H(\hat{\mathbf{B}}_2 \odot \mathbf{I}_N)\mathbf{f}\\ &+\sigma_{d_1}^2 - \sigma_{d_1}^2\gamma_2^{-1}\hat{\mathbf{b}}_3^H\mathbf{f} - \sigma_{d_1}^2\gamma_2^{-1}\mathbf{f}^H\hat{\mathbf{b}}_3 + \sigma_{z_2}^2\gamma_2^{-2}\hat{\mathbf{b}}_2\hat{\mathbf{b}}_2^H \end{aligned} \quad (3)$$

where $\mathbf{K}_1 = \mathbf{G}_2\mathbf{a}_2\mathbf{a}_2^H\mathbf{G}_2^H$, $\hat{\mathbf{K}}_2 = \mathbf{H}_1^H\hat{\mathbf{b}}_1^H\hat{\mathbf{b}}_1\mathbf{H}_1$, $\hat{\mathbf{K}}_3 = \mathbf{G}_2\mathbf{a}_2\hat{\mathbf{b}}_1\mathbf{H}_1$, $\hat{\mathbf{k}}_3 = \text{diag}(\hat{\mathbf{K}}_3^H)$, $\mathbf{B}_1 = \mathbf{G}_1\mathbf{a}_1\mathbf{a}_1^H\mathbf{G}_1^H$, $\hat{\mathbf{B}}_2 = \mathbf{H}_2^H\hat{\mathbf{b}}_2^H\hat{\mathbf{b}}_2\mathbf{H}_2$, $\hat{\mathbf{B}}_3 = \mathbf{G}_1\mathbf{a}_1\hat{\mathbf{b}}_2\mathbf{H}_2$, and $\hat{\mathbf{b}}_3 = \text{diag}(\hat{\mathbf{B}}_3^H)$. Here, $tr(\mathbf{F}\mathbf{A}\mathbf{F}^H) = \mathbf{f}^H(\mathbf{A} \odot \mathbf{I}_N)\mathbf{f}$, $tr(\mathbf{F}^H\mathbf{B}\mathbf{F}) = \mathbf{f}^H(\mathbf{B} \odot \mathbf{I}_N)\mathbf{f}$, $tr(\mathbf{F}^H\mathbf{A}\mathbf{F}\mathbf{B}) = \mathbf{f}^H(\mathbf{B}^T \odot \mathbf{A})\mathbf{f}$, $tr(\mathbf{F}\mathbf{A}\mathbf{F}^H\mathbf{B}) = \mathbf{f}^H(\mathbf{A}^T \odot \mathbf{B})\mathbf{f}$, $tr(\mathbf{A}^H\mathbf{F}^H) = \mathbf{f}^H\mathbf{a}$, and $tr(\mathbf{F}\mathbf{A}) = \mathbf{a}^H\mathbf{f}$ are used with $N \times N$ matrices $\mathbf{A}$ and $\mathbf{B}$, and an $N \times 1$ column vector $\mathbf{a}$. To determine the optimal $\mathbf{F}^\dagger, \mathbf{a}_1^\dagger, \mathbf{a}_2^\dagger, \mathbf{b}_1^\dagger$, and $\mathbf{b}_2^\dagger$ that minimize $\mathcal{J}(\mathbf{F})$ under the transmit power constraints at the two sources and the N relays, the Hamiltonian formulation [4] is written as

$$\begin{aligned}L(\mathbf{F}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2, \lambda_{s_1}, \lambda_{s_2}, \lambda_r) &= \mathcal{J}(\mathbf{F}) + \lambda_r(\mathbf{f}^H\mathbf{W}_1\mathbf{f} - p_r)\\ &+ \lambda_{s_1}(\sigma_{d_1}^2||\mathbf{a}_1||^2 - p_{s_1}) + \lambda_{s_2}(\sigma_{d_2}^2||\mathbf{a}_2||^2 - p_{s_2}) \end{aligned} \quad (4)$$

where $\mathbf{W}_1 = \sigma_{d_2}^2(\mathbf{K}_1 \odot \mathbf{I}_N) + \sigma_{d_1}^2(\mathbf{B}_1 \odot \mathbf{I}_N) + \sigma_{v_s}^2\mathbf{I}_N$, under the assumption that data symbols, channel coefficients, and noises are independent of each other. Here, λ_{s_1}, λ_{s_2}, and λ_r are Lagrangian multipliers. For expositional convenience, let $L(\mathbf{F}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2, \lambda_{s_1}, \lambda_{s_2}, \lambda_r)$ simply be denoted by $\mathcal{L}(\mathbf{F})$. Even though $\mathcal{L}(\mathbf{F})$ in (4) is not guaranteed to be jointly convex with respect to the variables $\{\mathbf{F}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}_1$ (or $\hat{\mathbf{b}}_1, \gamma_1), \mathbf{b}_2$ (or $\hat{\mathbf{b}}_2, \gamma_2)\}$, it is convex with respect to each of the variables [6]. In addition, $\mathcal{L}(\mathbf{F})$ in (4) is also convex over γ_1 and γ_2 [7]. Hence, alternating minimization procedures, where variables are optimized with respect to one at a time while keeping all others fixed, are applicable to obtaining a feasible local optimum [8] (see Section V). Refer to Table 1. The derivation is not simple. It is summarized in Theorem 1 as follows:

Theorem 1: Using the constrained Lagrangian optimization $\mathcal{L}(\mathbf{F})$ in (4), the optimal CSPDR solutions $\{\mathbf{f}^\dagger, \mathbf{a}_1^\dagger, \mathbf{a}_2^\dagger, \mathbf{b}_1^\dagger, \mathbf{b}_2^\dagger, \lambda_{s_1}^\dagger, \lambda_{s_2}^\dagger$, and $\lambda_r^\dagger\}$ for the two-way cooperative AF MIMO distributed relay network under the two-source power constraints and the relay power constraint can be written, respectively, as

$$\mathbf{f}^\dagger = \frac{\mathbf{W}_3^{-1}\mathbf{w}\sqrt{p_r}}{\sqrt{\mathbf{w}^H\mathbf{W}_3^{-H}\mathbf{W}_1\mathbf{W}_3^{-1}\mathbf{w}}} \quad (5)$$

$$\mathbf{a}_1^\dagger = \frac{\mathbf{T}_1^{-1}\boldsymbol{\phi}_1^H}{1 + \boldsymbol{\phi}_1\mathbf{T}_1^{-1}\boldsymbol{\phi}_1^H} \quad (6)$$

$$\mathbf{a}_2^\dagger = \frac{\mathbf{T}_2^{-1}\boldsymbol{\phi}_2^H}{1 + \boldsymbol{\phi}_2\mathbf{T}_2^{-1}\boldsymbol{\phi}_2^H} \quad (7)$$

$$\mathbf{b}_1^\dagger = \frac{\boldsymbol{\varphi}_1^H\mathbf{Z}_1^{-1}}{1 + \boldsymbol{\varphi}_1^H\mathbf{Z}_1^{-1}\boldsymbol{\varphi}_1} \quad (8)$$

$$\mathbf{b}_2^\dagger = \frac{\boldsymbol{\varphi}_2^H\mathbf{Z}_2^{-1}}{1 + \boldsymbol{\varphi}_2^H\mathbf{Z}_2^{-1}\boldsymbol{\varphi}_2} \quad (9)$$

$$\lambda_{s_1}^\dagger = \frac{\sigma_{v_s}^2||\mathbf{b}_2\mathbf{H}_2\mathbf{F}||^2}{p_{s_1}} + \frac{\sigma_{v_s}^2\sigma_{z_2}^2||\mathbf{b}_2||^2||\mathbf{H}_2\mathbf{F}||_F^2}{p_{s_1}p_r} \quad (10)$$

$$\lambda_{s_2}^\dagger = \frac{\sigma_{v_s}^2||\mathbf{b}_1\mathbf{H}_1\mathbf{F}||^2}{p_{s_2}} + \frac{\sigma_{v_s}^2\sigma_{z_1}^2||\mathbf{b}_1||^2||\mathbf{H}_1\mathbf{F}||_F^2}{p_{s_2}p_r} \quad (11)$$

$$\lambda_r^\dagger = \frac{\sigma_{z_1}^2||\mathbf{b}_1||^2 + \sigma_{z_2}^2||\mathbf{b}_2||^2}{p_r} \quad (12)$$

where $\mathbf{w} = \sigma_{d_2}^2\mathbf{k}_3 + \sigma_{d_1}^2\mathbf{b}_3$, $\mathbf{k}_3 = \text{diag}(\mathbf{K}_3^H)$, $\mathbf{K}_3 = \mathbf{G}_2\mathbf{a}_2\mathbf{b}_1\mathbf{H}_1$, $\mathbf{b}_3 = \text{diag}(\mathbf{B}_3^H)$, $\mathbf{B}_3 = \mathbf{G}_1\mathbf{a}_1\mathbf{b}_2\mathbf{H}_2$, $\mathbf{W}_3 = \mathbf{W}_2 + (\sigma_{z_1}^2||\mathbf{b}_1||^2 + \sigma_{z_2}^2||\mathbf{b}_2||^2){p_r}^{-1}\mathbf{W}_1$, $\mathbf{W}_2 = \sigma_{d_2}^2(\mathbf{K}_1^T \odot \mathbf{K}_2) + \sigma_{v_s}^2(\mathbf{K}_2 \odot \mathbf{I}_N) + \sigma_{d_1}^2(\mathbf{B}_1^T \odot \mathbf{B}_2) + \sigma_{v_s}^2(\mathbf{B}_2 \odot \mathbf{I}_N)$, $\mathbf{K}_2 = \mathbf{H}_1^H\mathbf{b}_1^H\mathbf{b}_1\mathbf{H}_1$, $\mathbf{B}_2 = \mathbf{H}_2^H\mathbf{b}_2^H\mathbf{b}_2\mathbf{H}_2$, $\mathbf{Z}_1 = \sigma_{v_s}^2\sigma_{d_2}^{-2}\mathbf{H}_1\mathbf{F}\mathbf{F}^H\mathbf{H}_1^H + \sigma_{z_1}^2\sigma_{d_2}^{-2}\mathbf{I}_M$, $\mathbf{Z}_2 = \sigma_{v_s}^2\sigma_{d_1}^{-2}\mathbf{H}_2\mathbf{F}\mathbf{F}^H\mathbf{H}_2^H + \sigma_{z_2}^2\sigma_{d_1}^{-2}\mathbf{I}_M$, $\mathbf{T}_1 = \lambda_r\mathbf{G}_1^H\mathbf{F}^H\mathbf{F}\mathbf{G}_1 + \lambda_{s_1}\mathbf{I}_M$, $\mathbf{T}_2 = \lambda_r\mathbf{G}_2^H\mathbf{F}^H\mathbf{F}\mathbf{G}_2 + \lambda_{s_2}\mathbf{I}_M$, $\boldsymbol{\phi}_1 = \mathbf{b}_2\mathbf{H}_2\mathbf{F}\mathbf{G}_1$, $\boldsymbol{\phi}_2 = \mathbf{b}_1\mathbf{H}_1\mathbf{F}\mathbf{G}_2$, $\boldsymbol{\varphi}_1 = \mathbf{H}_1\mathbf{F}\mathbf{G}_2\mathbf{a}_2$, and $\boldsymbol{\varphi}_2 = \mathbf{H}_2\mathbf{F}\mathbf{G}_1\mathbf{a}_1$. Here, using the vector form $\mathbf{f}^\dagger$ in (5), the corresponding optimal relay amplifying matrix $\mathbf{F}^\dagger$ can be written as $\mathbf{F}^\dagger = \frac{\text{diag}(\mathbf{W}_3^{-1}\mathbf{w})\sqrt{p_r}}{\sqrt{\mathbf{w}^H\mathbf{W}_3^{-H}\mathbf{W}_1\mathbf{W}_3^{-1}\mathbf{w}}}$.

Proof: Refer to [9]. ■

Using (8) and (9), the optimal scaling factors $(\gamma_1^\dagger)^{-1}$ and $(\gamma_2^\dagger)^{-1}$ can be obtained, respectively, as

$$(\gamma_1^\dagger)^{-1} = \frac{\sigma_{d_2}^2}{\hat{p}_{y_2}}\left(\frac{\vartheta_2}{1 + \vartheta_2}\right) \text{ and } (\gamma_2^\dagger)^{-1} = \frac{\sigma_{d_1}^2}{\hat{p}_{y_1}}\left(\frac{\vartheta_1}{1 + \vartheta_1}\right) \quad (13)$$

where $\vartheta_1 = \hat{\boldsymbol{\phi}}_1\mathbf{T}_1^{-1}\hat{\boldsymbol{\phi}}_1^H > 0$ and $\vartheta_2 = \hat{\boldsymbol{\phi}}_2\mathbf{T}_2^{-1}\hat{\boldsymbol{\phi}}_2^H > 0$.

Using (6) and the generalized Rayleigh quotient [10], the maximum SNR_1 at $\mathbb{D}_1$ and the maximum SNR_2 at $\mathbb{D}_2$ over $\mathbf{f}$ are written, respectively, as

$$\begin{aligned}\max_{\mathbf{f}}\left[\text{SNR}_1 = \frac{\sigma_{d_2}^2|\mathbf{b}_1\mathbf{H}_1\mathbf{F}\mathbf{G}_2\mathbf{a}_2|^2}{\sigma_{v_s}^2||\mathbf{b}_1\mathbf{H}_1\mathbf{F}||^2 + \sigma_{z_1}^2||\mathbf{b}_1||^2} = \frac{\mathbf{f}^H\boldsymbol{\Phi}_1\mathbf{f}}{\mathbf{f}^H\boldsymbol{\Gamma}_1\mathbf{f}}\right]\\ = \mathbf{eig}_{\max}(\boldsymbol{\Gamma}_1^{-1}\boldsymbol{\Phi}_1) = \psi_1 \quad (14)\end{aligned}$$

$$\max_{\mathbf{f}}\left[\text{SNR}_2 = \frac{\sigma_{d_1}^2|\mathbf{b}_2\mathbf{H}_2\mathbf{F}\mathbf{G}_1\mathbf{a}_1|^2}{\sigma_{v_s}^2||\mathbf{b}_2\mathbf{H}_2\mathbf{F}||^2 + \sigma_{z_2}^2||\mathbf{b}_2||^2} = \frac{\mathbf{f}^H\boldsymbol{\Phi}_2\mathbf{f}}{\mathbf{f}^H\boldsymbol{\Gamma}_2\mathbf{f}}\right]$$

$$= \mathbf{eig}_{\max}\big(\mathbf{\Gamma}_2^{-1}\mathbf{\Phi}_2\big) = \psi_2 \tag{15}$$

where $\mathbf{\Gamma}_1 = \mathbf{K}_2 \odot \mathbf{I}_N + \sigma_{z_1}^2 p_r^{-1}||\mathbf{b}_1||^2\mathbf{W}_1$, $\mathbf{\Phi}_1 = \sigma_{d_2}^2\mathbf{K}_1^T \odot \mathbf{K}_2$, $\mathbf{\Gamma}_2 = \mathbf{B}_2 \odot \mathbf{I}_N + \sigma_{z_2}^2 p_r^{-1}||\mathbf{b}_2||^2\mathbf{W}_1$, and $\mathbf{\Phi}_2 = \sigma_{d_1}^2\mathbf{B}_1^T \odot \mathbf{B}_2$. Here, $\mathbf{eig}_{\max}\big(\mathbf{\Gamma}_1^{-1}\mathbf{\Phi}_1\big)$ and $\mathbf{eig}_{\max}\big(\mathbf{\Gamma}_2^{-1}\mathbf{\Phi}_2\big)$ are maximum eigenvalues of $\mathbf{\Gamma}_1^{-1}\mathbf{\Phi}_1$ and $\mathbf{\Gamma}_2^{-1}\mathbf{\Phi}_2$, respectively. Additionally, both $\mathbf{\Gamma}_1^{-1}\mathbf{\Phi}_1$ and $\mathbf{\Gamma}_2^{-1}\mathbf{\Phi}_2$ have only one positive eigenvalue, which is the maximum eigenvalue, because their ranks are 1, regardless of N and M. Using (14) and (15), the sum of the achievable rate $\mathcal{R}$ of the two-way AF MIMO beamforming distributed relay network under the transmit power constraints at two sources and relays can be written as

$$\begin{aligned}\mathcal{R} &= \frac{1}{2}\log_2\big(1+\max \mathrm{SNR}_1\big) + \frac{1}{2}\log_2\big(1+\max \mathrm{SNR}_2\big)\\ &= \frac{1}{2}\log_2\big(1+\psi_1+\psi_2+\psi_1\psi_2\big).\end{aligned} \tag{16}$$

Using the optimal solutions of $\mathcal{L}(\mathbf{F})$, the $\mathcal{J}(\mathbf{F}^\dagger)$ in (12) can be rewritten as

$$\begin{aligned}\mathcal{J}(\mathbf{F}^\dagger) &= \sigma_{d_2}^2(1-\boldsymbol{\varphi}_1^H(\mathbf{b}_1^\dagger)^H) + \sigma_{d_1}^2(1-\boldsymbol{\varphi}_2^H(\mathbf{b}_2^\dagger)^H))\\ &= \frac{\sigma_{d_2}^2}{1+\psi_1} + \frac{\sigma_{d_1}^2}{1+\psi_2}.\end{aligned} \tag{17}$$

Additionally, using (16) and (17), the MMSE cost function with the unit power of the transmitted signals at $\mathbb{S}_1$ and $\mathbb{S}_2$ can be rewritten as

$$\mathcal{J}(\mathbf{F}) = 2^{-2\mathcal{R}}(2+\psi_1+\psi_2). \tag{18}$$

If $\mathbb{S}_1$ and $\mathbb{S}_2$ have knowledge of backward and forward channels, the EB can be employed at $\mathbb{S}_1$ and $\mathbb{S}_2$ (or $\mathbb{D}_1$ and $\mathbb{D}_2$), respectively, as

$$\mathbf{a}_1 = \mathbf{v}_1,\ \mathbf{a}_2 = \mathbf{v}_2,\ \mathbf{b}_1 = \mathbf{u}_1,\ \text{and } \mathbf{b}_2 = \mathbf{u}_2 \tag{19}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the right singular vectors corresponding to the largest singular value of $\mathbf{G}_1$ and $\mathbf{G}_2$, and $\mathbf{u}_1$ and $\mathbf{u}_2$ are the left singular vectors corresponding to the largest singular value of $\mathbf{H}_1$ and $\mathbf{H}_2$, respectively. These may not be optimal as the ones in (6)-(9). The EB vectors at $\mathbb{S}_1$ and $\mathbb{S}_2$ satisfy the power constraints at $\mathbb{S}_1$ and $\mathbb{S}_2$ because a singular vector is a unit norm vector.

Theorem 2: Using (19), the explicitly optimal $\mathbf{f}^\dagger$, the optimal equalizer coefficient $\alpha^\dagger$, and the optimal Lagrangian multiplier $\lambda_r^\dagger$ for the two-way AF MIMO relay network under the relay transmit power constraint with EB vectors can be written, respectively, as

$$\mathbf{f}^\dagger = \frac{\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2}{\sqrt{(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)^H\mathbf{H}(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)}}\sqrt{p_r} \tag{20}$$

$$\alpha^\dagger = \frac{\sqrt{p_r}}{\sqrt{(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)^H\mathbf{H}(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)}} \tag{21}$$

$$\lambda_r^\dagger = \frac{(\sigma_{z_1}^2+\sigma_{z_2}^2)\big|\big|(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)^H\mathbf{H}^{1/2}\big|\big|}{p_r^2} \tag{22}$$

where $\mathbf{H} = \sigma_{d_1}^2\mathbf{Q}_1^H\mathbf{Q}_1 + \sigma_{d_2}^2\mathbf{Q}_2^H\mathbf{Q}_2 + \sigma_{v_s}^2\mathbf{I}_N$, $\boldsymbol{\varpi}_1 = \frac{\sigma_{d_1}^2\mathbf{W}_4^{-1}\mathbf{Q}_1^H\mathbf{d}_2^H}{1+\sigma_{d_1}^2\mathbf{d}_2\mathbf{Q}_1\mathbf{W}_4^{-1}\mathbf{Q}_1^H\mathbf{d}_2^H}$, $\boldsymbol{\varpi}_2 = \frac{\sigma_{d_2}^2\mathbf{W}_5^{-1}\mathbf{Q}_2^H\mathbf{d}_1^H}{1+\sigma_{d_2}^2\mathbf{d}_1\mathbf{Q}_2\mathbf{W}_5^{-1}\mathbf{H}_2^H\mathbf{d}_1^H}$, $\mathbf{W}_4 = \sigma_{d_2}^2\mathbf{Q}_2^H\mathbf{d}_1^H\mathbf{d}_1\mathbf{Q}_2 + \mathbf{W}_6$, $\mathbf{W}_5 = \sigma_{d_1}^2\mathbf{Q}_1^H\mathbf{d}_2^H\mathbf{d}_2\mathbf{Q}_1 + \mathbf{W}_6$, and $\mathbf{W}_6 = \sigma_{v_s}^2(\mathbf{D}_1^H\mathbf{D}_1 + \mathbf{D}_2^H\mathbf{D}_2) + p_r^{-1}(\sigma_{z_1}^2+\sigma_{z_2}^2)\mathbf{H}$. Here, $\mathbf{q}_1 = \mathbf{G}_1\mathbf{v}_1$, $\mathbf{q}_2 = \mathbf{G}_2\mathbf{v}_2$, $\mathbf{d}_1 = \mathbf{u}_1\mathbf{H}_1$, $\mathbf{d}_2 = \mathbf{u}_2\mathbf{H}_2$, $\mathbf{Q}_1 = \mathrm{diag}(\mathbf{q}_1)$, $\mathbf{Q}_2 = \mathrm{diag}(\mathbf{q}_2)$, $\mathbf{D}_1 = \mathrm{diag}(\mathbf{d}_1)$, and $\mathbf{D}_2 = \mathrm{diag}(\mathbf{d}_2)$ are used. Also, using the vector form $\mathbf{f}^\dagger$ in (20), the corresponding optimal relay amplifying matrix $\mathbf{F}^\dagger$ with EB vectors can be written as $\mathbf{F}^\dagger = \frac{\mathrm{diag}(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)\sqrt{p_r}}{\sqrt{(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)^H\mathbf{H}(\boldsymbol{\varpi}_1+\boldsymbol{\varpi}_2)}}$.

Proof: Refer to [9]. ■

TABLE I
ITERATIVE ALGORITHM PROCEDURE

Step 1	Initialization: $k = 0$ $\mathbf{f}(0) = [1,\cdots,1]^T, \mathbf{a}_1(0) = [1,1]^T, \mathbf{a}_2(0) = [1,1]^T, \mathcal{J}(\mathbf{F}(0)) = 2$
Step 2	Iteration: $k \rightarrow k+1$ $\mathbf{b}_1(k) = f_{\mathbf{b}_1}(\mathbf{F}(k\text{-}1), \mathbf{a}_2(k\text{-}1))$, $\mathbf{b}_2(k) = f_{\mathbf{b}_1}(\mathbf{F}(k\text{-}1), \mathbf{a}_1(k\text{-}1))$, $\mathbf{a}_1(k) = f_{\mathbf{a}_1}(\mathbf{F}(k\text{-}1), \mathbf{b}_2(k))$, $\mathbf{a}_2(k) = f_{\mathbf{a}_2}(\mathbf{F}(k\text{-}1), \mathbf{b}_1(k))$, $\mathbf{F}(k) = f_{\mathbf{F}}(\mathbf{a}_1(k), \mathbf{a}_2(k), \mathbf{b}_1(k), \mathbf{b}_2(k))$, $\mathcal{J}(\mathbf{F}(k)) = f_{\mathcal{J}(\mathbf{F}(k))}(\mathbf{F}(k), \mathbf{a}_1(k), \mathbf{a}_2(k), \mathbf{b}_1(k), \mathbf{b}_2(k))$
Step 3	If $\lvert\mathcal{J}(\mathbf{F}(k\text{-}1)) - \mathcal{J}(\mathbf{F}(k))\rvert \leq \eta$ or $k \geq \xi$ go to Step 4 and stop, otherwise go back to Step 2 ($\eta = 10^{-4}$ and $\xi = 50$)
Step 4	$\mathbf{a}_1 = \mathbf{a}_1(k);\ \mathbf{a}_2 = \mathbf{a}_2(k);\ \mathbf{b}_1 = \mathbf{b}_1(k);\ \mathbf{b}_2 = \mathbf{b}_2(k);\ \mathbf{F} = \mathbf{F}(k)$

IV. SIMULATION RESULTS AND DISCUSSIONS

The two sources employ quadrature phase-shift keying with unity power. All complex backward and forward channel matrixes $\mathbf{G}_1$, $\mathbf{G}_2$, $\mathbf{H}_1$, and $\mathbf{H}_2$ are i.i.d. Gaussian random variables. They are invariant during data transmission over a frame of 5×10^5 data symbols. Then, they are newly generated over the next frame. The transmit power constraints at the two sources and relays are set to $p_{s_1} = p_{s_2} = 1$ and $p_r = 1$, respectively. Assume that all nodes have the same thermal noise power, i.e., $\sigma_{v_s}^2 = \sigma_{z_1}^2 = \sigma_{z_2}^2$.

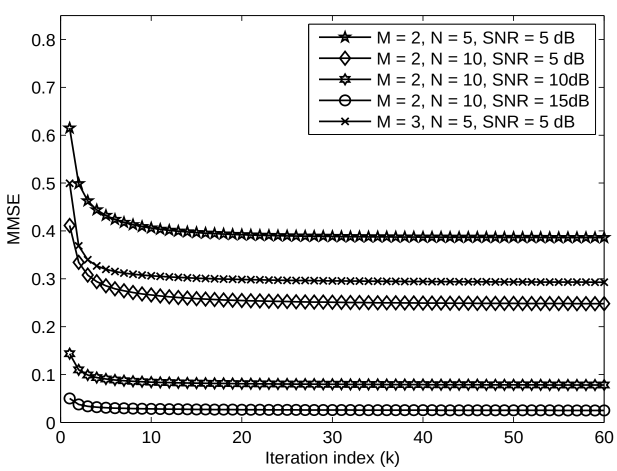

Fig. 2. MMSE versus iteration index k showing convergence of iterative algorithm in Table I for $M = 2$ and $N = 5, 10$ with input SNR = 5, 10, and 15 dB.

Figure 2 shows numerically the convergence property of the proposed iterative algorithm for the CSPDR with $M = 2$ source transmitter antennas; an input SNR of 5, 10, and 15 dB; and a different number of relays $N = 5, 10$. Observe that the algorithm converges within the first 10 iterations.

Figure 3 shows the MMSE versus the number of relays N using the optimal solutions in (5)-(9) and the optimal $\mathcal{J}(\mathbf{F}^\dagger)$

in (17) with the input SNR = 3, 6, and 9 dB for two cases: case (1) for a fixed $M = 4$ case with N varying from 2 to 12, and case (2) for a fixed $N = 4$ case with M varying from 2 to 12. Observe that when $M > N$, a smaller MMSE is observed than the case of $N > M$ for a given SNR. This implies that the effect of beamforming vectors at the sources becomes more dominant than that of the relay amplifying matrix at the relay. Analytically, when $M > N$, $\mathbf{eig}_{\max}(\mathbf{\Gamma}_1^{-1}\mathbf{\Phi}_1)$ and $\mathbf{eig}_{\max}(\mathbf{\Gamma}_2^{-1}\mathbf{\Phi}_2)$ (i.e., ψ_1 and ψ_2) are greater than the ones with $M < N$. As a result, the smaller MMSE is observed when $M > N$. Observe also that the optimal MMSE cost function $(\mathcal{J}(\mathbf{F}^{\dagger}))$ values are less than 2 at all times, regardless of N and SNR, which can be proven with (17). Finally, the crossing point in Fig. 3 is for the case of $M = N$, where $\mathbf{eig}_{\max}(\mathbf{\Gamma}_1^{-1}\mathbf{\Phi}_1)$ and $\mathbf{eig}_{\max}(\mathbf{\Gamma}_2^{-1}\mathbf{\Phi}_2)$ are equal.

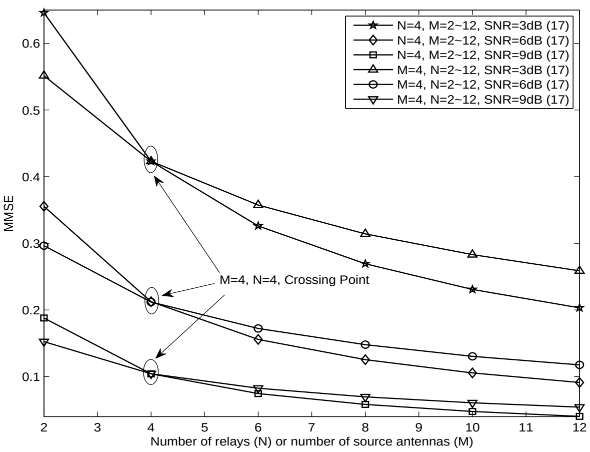

Fig. 3. MMSE versus number of relays N and number of source antennas M using optimal solutions in (5)-(9) and optimal $\mathcal{J}(\mathbf{F}^{\dagger})$ in (17). Input SNR was 3, 6, and 9 dB and two cases were considered: case (1) for a fixed $M = 4$ case with N varying from 2 to 12, and case (2) for a fixed $N = 4$ case with M varying from 2 to 12.

Figure 4 shows the SER performance versus input SNR using the optimal and simplified relay amplifying matrices in (5) and (20), respectively, for $M = 2$ and $N = 3$. SER results for the existing schemes in [1]–[3] are also presented for $M = 1, 2$ and $N = 3$ for comparison. In addition, the reciprocal channels are used in Fig. 4, as done in [1]. Observe in Fig. 4 that the proposed scheme in (14) shows better performance (e.g., 0.5 dB better at SER $= 10^{-6}$) than the one in [1] for a high SNR but worse for a low SNR. This difference is due to the normalization issue, which will be explained in Fig. 5. Additionally, observe that the proposed iterative optimal solution in (5) shows about 2 dB better SER performance at SER $= 10^{-3}$ than the simplified EB vectors in (20). Furthermore, observe that even the simplified SER performance in (20) shows better performance (e.g., 4.5 dB at SER $= 10^{-3}$) than the existing one in [2]. This is because the solutions in [2] are nonoptimal. Furthermore, the proposed scheme with $M = 1$ and $N = 3$ shows a slightly better performance (e.g., 0.6 dB at SER $= 10^{-3}$) than the existing one in [3] under the same environment. This is because the transmit power is constrained at the relay only in [3], whereas in this paper, both source and relay transmit powers are constrained. In summary, the optimal solutions in (5)-(9) of this paper show the best SER performance among all schemes.

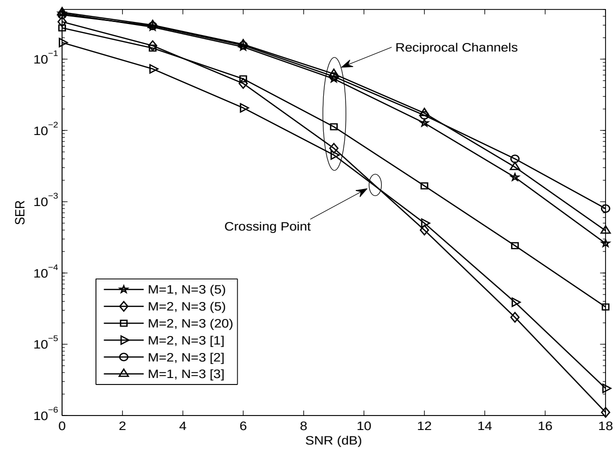

Fig. 4. SER performance versus input SNR using different optimal relay amplifying matrices in (5) and (20) for $M = 2$ and $N = 3$. Existing schemes in [1]–[3] are also presented for $M = 1, 2$ and $N = 3$.

Remark 1 : The scheme in [2] is close to the one proposed in this paper. The relay amplifying matrix was designed in [2] as follows: First, the derivative of the cost function in (13) of [2] was taken with respect to a nondiagonal relay amplifying matrix. Then, after obtaining the nondiagonal relay amplifying matrix solution, only its diagonal elements were used in (17) of [2]. On the other hand, in this paper, a diagonal relay amplifying matrix is assumed first in (3) before the optimization, due to the relays' dispersed location, and then the MMSE criterion (or Karush-Kuhn-Tucker) is applied using the property of the Hadamard product. Then, a derivative is taken with respect to a diagonal relay amplifying matrix. Due to this difference, the SER performance with the proposed optimal diagonal relay amplifying matrix outperforms the existing one in [2] as shown in Fig. 4, e.g., 6.5 dB gain at SER $= 10^{-3}$. Additionally, in Fig. 4, the proposed two-way AF SISO wireless relay system (i.e., $M = 1$) is compared with the existing one in [3]. For example, observe that 0.6 dB at SER $= 10^{-3}$ is better than the one in [3].

Remark 2 : The two-way relay network system model in this current paper is different from the one used in [1] from three perspectives. The first difference is that this current paper aims to find the three optimum parameters iteratively: (a) optimum relay amplifying matrix $\mathbf{F}$, (b) optimum transmit beamforming vector $\mathbf{a}_i$ ($\mathbf{u}_i$ in [1]), and (c) optimum receive beamforming vector $\mathbf{b}_j$ ($\mathbf{v}_j$ in [1], $i \neq j$). On the other hand, the authors in [1] found $\mathbf{a}_i$ and $\mathbf{b}_j$ but not $\mathbf{F}$. The second difference is that the MMSE criteria was employed in the proposed one, while the maximum ratio transmission principle in reference [7] of [1] and the principle eigenvector of the eigenvalue were employed in [1]. For a fair comparison, in Fig. 4, we generate the same transmit beamforming vector

$\mathbf{a}_i$ and receive beamforming vector $\mathbf{b}_j$ using the maximum ratio transmission principle and the principle eigenvector of the eigenvalue, respectively, with (4) and (6) in [1]. Also, we employ $\mathbf{F} = \mathbf{I}$, an identity matrix, to simulate the unit gain relay amplifying matrix. The last difference is that the one in [1] uses one single relay of N multiple antenna elements, but this paper employs N dispersed N relays, each with a single antenna element. The same Rayleigh fading channel model is used for a fair comparison.

Figure 5 shows the SER performance and the relay transmit power versus input SNR using (5) and the existing one in [1] with/without normalization of the relay transmit power for $M = 2$ and $N = 3$. Simulation results indicate that the transmit power p_r at the relay in [1] can be greater than one, e.g., $p_r = 9 \sim 12$, when $M = 2$ and $N = 3$. This relay transmit power in [1] is significantly higher than $p_r = 1$ in the proposed scheme. This is why the proposed scheme shows a worse performance than the one in [1] at a low SNR, as shown in Fig. 4. When the relay transmit power in [1] is normalized as the proposed scheme in this paper for a fair comparison, then the proposed scheme shows much better performance (e.g., 4.1 dB at SER $= 10^{-2}$) than the one in [1] for both low and high SNRs, as shown in Fig. 5. The difference between the two results becomes larger as either the input SNR or N increases.

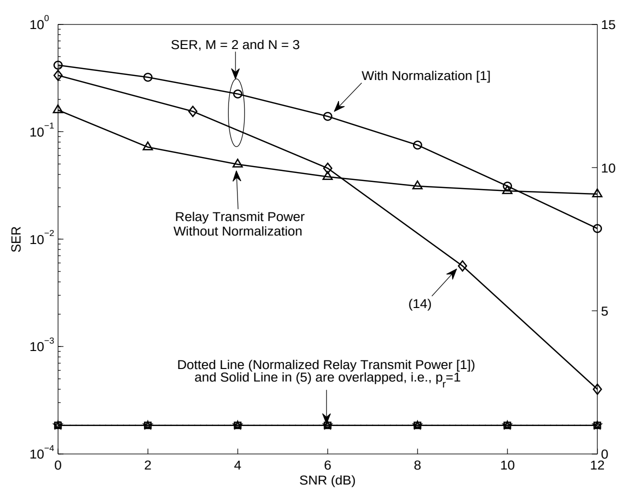

Fig. 5. SER performance and transmit relay power versus input SNR using (5) and the existing one in [1] with/without normalization of relay transmit power for $M = 2$ and $N = 3$.

Figure 6 shows the sum of the achievable rate $\mathcal{R}$ versus the number of relays N and the number of source antennas M with input SNR = 3, 6 dB using the optimum and the simplified relay amplifying matrices in (5) and (20) for the two cases: case (1) $M = 2$ with N varying from 2 to 12, and case (2) $N = 2$ with M varying from 2 to 12, respectively. The higher $\mathcal{R}$ is observed with the optimum relay amplifying matrix and the optimum transmit/receive beamforming vectors in (5)-(9) than with the simplified relay amplifying matrix and the EB vectors in (20).

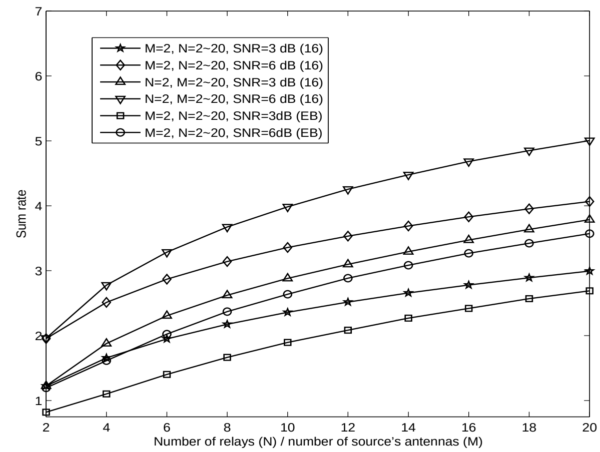

Fig. 6. Sum rate versus number of relays *N* and number of source antennas M with input SNR = 3, 6 dB using different relay amplifying matrices in (5) and (20) for two cases: case (1) $M = 2$ with N varying from 2 to 12, and case (2) $N = 2$ with M varying from 2 to 12.

V. Conclusion

This paper presented an MMSE-based optimum relay amplifying matrix for the single-antenna multiple dispersed relays and optimum transmit/receive beamforming vectors for the multiple-antenna sources explicitly and iteratively in the two-way AF MIMO relay network under transmit power constraints at the two sources and relays. This paper also presented a simplified relay amplifying matrix with EB vectors. It was observed that both the proposed optimum and simplified schemes show significantly better performance than the existing ones in [1]–[3] under the same environments. Finally, the MMSE cost function values are less than 2 at all times, regardless of the number of relays. The two-way AF wireless relay system proposed in this paper can be applicable in a future relay network with relays of CRAN to improve spectral efficiency.

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